$\vec{E}$ field in the cover region is expressed as

$$\vec{E} = -j\omega\mu\int_S \vec{G}\cdot\vec{K}dS - \nabla(1/\epsilon)$$
$$\cdot\left[\oint_{C_0} G\rho_{10}\,dl' + \oint_{C_p} G\rho_{1p}\,dl' + \int_S G\sigma\,dS'\right] \quad (11)$$

where $G = G^p + G^r$ is a Sommerfeld integral representation of the scalar potential Green's function for layered media. The gradient operator may (at interior points) be exchanged with the spectral integrals without rendering those Sommerfeld integrals nonconvergent.

V. Summary and Conclusions

The preceding sections illustrate a procedure for avoiding the needless imposition of derivatives onto the Green's function, thus avoiding convergence problems. In the process of converting to a less singular formulation, unknown surface and line charges are introduced explicitly into the problem. Presumably, for many applied problems, these charge functions could be expanded in suitable moment method basis sets along with the original surface currents.

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Dominance of Resistive Losses over Hysteretic Losses in Ferromagnetic Conductors

K. K. THORNBER

***Abstract*—In the presence of ferromagnetic conductors, electromagnetic signal propagation might be seriously degraded through hysteretic losses. It is shown, however, that, independent of frequency, such losses are always smaller than the ordinary resistive losses one would calculate for a similarly resistive, equivalent paramagnetic material. This indicates that measurements of conductivity and permeability suffice to bound hysteretic losses, obviating the necessity for measurements of hysteresis at operating frequencies.**

I. Introduction

Electrical interconnection within novel, multichip packaging technologies [1] can require magnetic materials in order to satisfy the constraints of processing. Thus, for example, nickel can be used as a vapor barrier protecting an underlying copper layer against the products and reactants present during the curing of an adjacent polyimide insulating layer. Being capable of electroless deposition, Ni can also be used for thick, vertical structures such as vias. Other magnetic materials might also be included to meet special objectives.

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The author is with AT&T Bell Laboratories, Murray Hill, NJ 07974.
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Whenever magnetic materials are used in an electromagnetic environment, hysteretic losses are induced by the time-dependent fields [2]. Such losses are difficult to measure and to calculate. For example, a calculation requires knowledge of $B(x,t)$ given $H(x,t')$ for $t' < t$, under spatially nonuniform conditions, i.e., field strengths and hence hysteresis that are different in each portion of the material. Boundary conditions introduce additional complications.

It is the purpose of this short note to observe that under rather general conditions, the actual power losses to magnetic hysteresis in conducting materials are less than power losses to electrical resistance, the latter being calculated for a situation in which locally $B(x,\omega) = \mu_e(\omega)H(x,\omega)$ and $J(x,\omega) = \sigma(\omega)E(x,\omega)$. For the purposes of the calculation of the resistive losses, the material is conductive and either paramagnetic or diamagnetic. Since the conductivity, σ, and the permeability, μ_e, are readily measurable, this approach for bounding hysteretic losses can be implemented directly.

II. Derivation

That resistive losses exceed hysteretic losses is derived as follows. Starting with Ampere's law at the frequency of interest (ignoring the displacement currents),

$$\nabla \times H = \sigma E \quad (1)$$

and taking the direction of penetration into the magnetic material to be $\hat{x}$, it follows that the largest transverse components of the fields are related by

$$\sigma E_z(x) = \delta^{-1}H_y(x) \quad (2)$$

where $\delta \equiv (-i\omega\mu_e\sigma)^{-1/2}$, the complex skin depth. The skin depth can be obtained by combining Gauss' law,

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3)$$

with (1) to yield

$$\nabla^2 H = \frac{\sigma\partial B}{\partial t} = -i\omega\sigma\mu_e H. \quad (4)$$

In each volume element, therefore, the resistive power dissipation $\sigma|E|^2/2$ becomes

$$P_E(x) = \frac{\sigma|E|^2}{2} = \frac{|H|^2}{(2|\delta|^2\sigma)} = \pi\nu\mu_e|H|^2 \quad (5)$$

where $\nu = \omega/2\pi$ is the frequency of interest. P_E is thus the local rate of energy dissipation per unit volume which would be calculated for a paramagnetic material of effective permeability μ_e.

The actual hysteretic loss at x per unit volume is given by

$$P_H(x) = \nu\oint H(x)\cdot dB(x) \quad (6)$$

which under saturation conditions can be expressed in terms of H_c, H_s, and B_s as defined in Fig. 1 as

$$P_H(x) = 4\nu H_c(x)B_s(x). \quad (7)$$

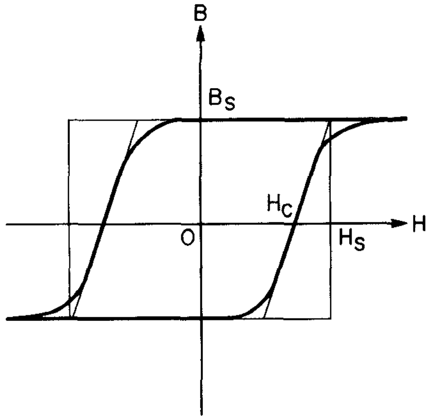

Fig. 1. Simple representation of ferromagnetic hysteresis for the purposes of estimating power losses. Treating the material in an equivalent manner paramagnetically ($\mu_e \equiv B_s/H_s$), the resistive losses would be represented by 80 percent ($\pi/4$) of the $H_s - B_s$ rectangle. The actual hysteretic losses are, of course, represented by the $H_c - B_s$ parallelogram. (Increasing μ_e, the effective paramagnetic permeability, to $1.3B_s/H_s$, the equivalent resistive losses would be 100 percent of the $H_s - B_s$ rectangle.)

Comparing with (5) for $H = H_s$,

$$P_E(x) = \pi \nu H_s(x) \mu_e H_s(x). \tag{8}$$

We note that so long as

$$\mu_e = \frac{B_s}{H_s} \leqslant \frac{\partial B}{\partial H}(\max) = \mu_m \tag{9}$$

then

$$P_H \leqslant P_E \tag{10}$$

as we desired to show. (This will be true so long as $H_c \leq \pi H_s/4$, which is usually the case. $H_c \leq H_s$ is always true.) Note that (10) is true for $H > H_s$, since the ohmic losses continue to increase linearly with $|H|^2$ while the hysteretic losses clearly saturate. For $H < H_s$, (10) is also valid since the size of (6) reduces approximately as H^2, any deviation being absorbed into μ_e as the latter approaches $\partial B/\partial H(\max)$. Finally, if in (4) H should vary spatially more rapidly than its skin-depth-induced variation, due for example to geometrical constraints on the spatial extent of H, ohmic losses are enhanced. For example, an $\exp(ik_y y + ik_z z)$ dependence in H results in an *enhanced* ohmic loss P_E', related to P_E according to

$$P_E' = \left| \frac{k_y^2 + k_z^2 - i\omega\sigma\mu}{-i\omega\sigma\mu} \right| P_E. \tag{11}$$

Here, as a specific example suggested by a referee, consider a cylindrical conductor of radius a carrying a uniform current J at low frequency. One finds

$$P_E' = \frac{2|H(r)|^2}{\sigma r}$$

for the resistive power at radius $r \leqslant a$. Comparing with (5), we note that δ has been superseded by $r/2 \ll \delta$, as suggested by (11). Thus $P_E' > P_E > P_H$, and the dominance of resistive losses is manifest. Should the material be paramagnetic, $H_c = 0$, hence $P_H = 0$. Based on (10) and (11) for a given measured σ and μ_e and treating the ferromagnetic conductor as paramagnetic, that is, as having $B = \mu_e H$ for constant μ_e, resistive power dissipation exceeds any hysteretic loss in the actual material at any operating frequency.

Strictly speaking, we have so far only compared the actual hysteretic losses of a ferromagnetic conductor (Fig. 1) with the ordinary resistive losses in an equivalent paramagnetic material. While this is our principal result (it permits an easily calculated upper bound to dissipation due to hysteresis), one naturally asks about the actual resistive losses in the ferromagnetic conductor. Consider two limiting cases. The first is that of nearly paramagnetic behavior ($H_c \ll H_s$): In this case our linear equivalent model is close to the actual material; thus (10) follows at once. The second case is that of hard ferromagnetic (HF) behavior ($H_c \approx H_s$). In this case

$$P_{E,\mathrm{HF}} \approx 4\nu H_s(x) B_s(x) \left(2^{1/2} \omega\sigma\mu_m(x) |\delta_f(x)|^2 \right) \tag{12}$$

where δ_f is an effective ferromagnetic skin depth satisfying

$$|\delta_f| \leqslant (\omega\sigma\mu_m)^{-1/2}. \tag{13}$$

Hence $P_{E,\mathrm{HF}} \geqslant P_E \geqslant P_H$ from (7), (10), (12), and (13). These two limiting cases strongly suggest, but of course do not prove, that for intermediate cases as well, resistive losses do indeed dominate hysteretic losses, as claimed by the title. Finally, as indicated in (11), geometric constraints again enhance the ohmic losses, further strengthening the dominance.

III. Implications

The above result indicates that measurements of conductivity and permeability suffice to estimate hysteretic losses, obviating the necessity for measurements of hysteresis at the frequencies of interest. Low-frequency, metallic conductivities are usually accurate to frequencies associated with the far infrared (1 THz), while permeabilities usually decrease with higher frequency as the magnetic domains cease to follow changes in the fields. As the latter effect further reduces the hysteretic loss, it is apparent that difficult, high-frequency hysteresis experiments need *not* be undertaken for purposes of estimating *total* power loss.

It should be noted that we have considered only *conducting* ferromagnets. For *insulating* ferromagnets, such as ferrites, the relations between the electric and magnetic fields are more complicated: full hysteresis curves at the frequency of interest would seem to be required to adequately estimate power dissipation. Finally, although this work was motivated by concerns for signal propagation in multichip packaging technology, the results clearly apply whenever magnetic materials are penetrated by electromagnetic fields.

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